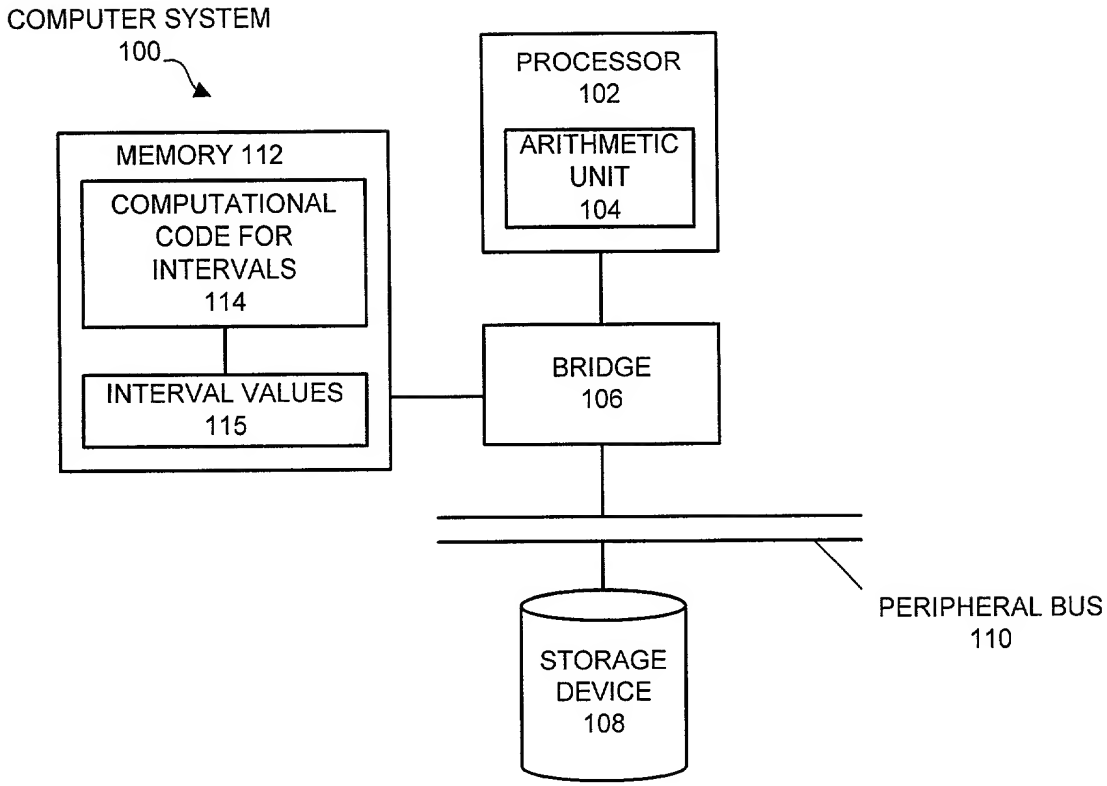
COMPUTER SYSTEM
100
PROCESSOR
102
ARITHMETIC
UNIT
104
MEMORY 112
COMPUTATIONAL
CODE FOR
INTERVALS
114
INTERVAL VALUES
115
BRIDGE
106
PERIPHERAL BUS
110
STORAGE
DEVICE
108

FIG. 1

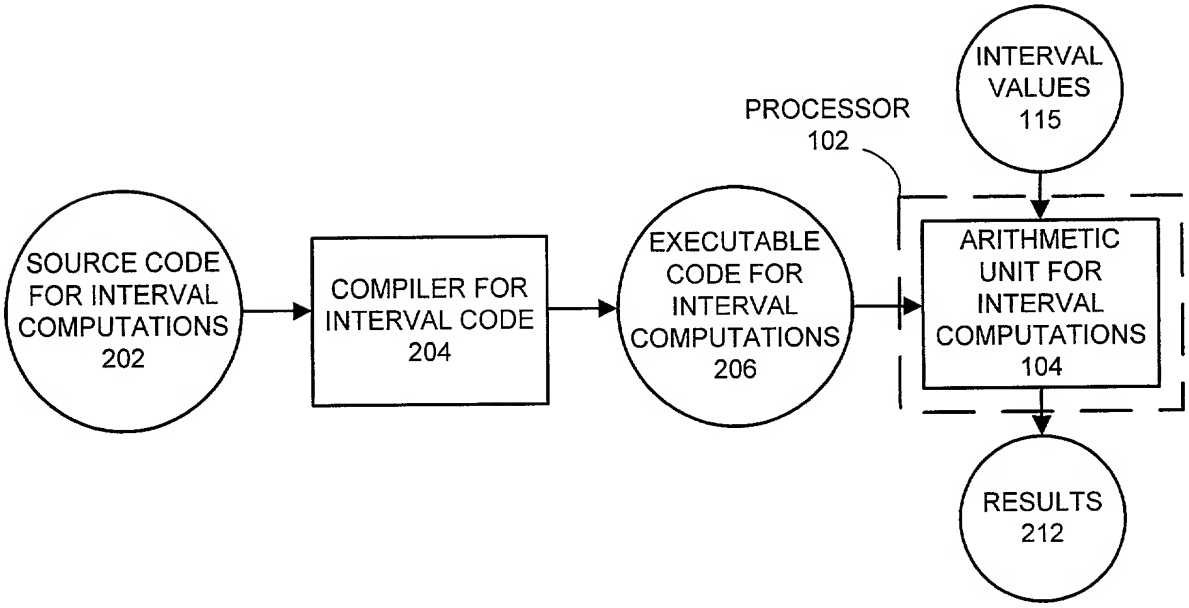
INTERVAL
VALUES
115
PROCESSOR
102
SOURCE CODE
FOR INTERVAL
COMPUTATIONS
202
COMPILER FOR
INTERVAL CODE
204
EXECUTABLE
CODE FOR
INTERVAL
COMPUTATIONS
206
ARITHMETIC
UNIT FOR
INTERVAL
COMPUTATIONS
104
RESULTS
212

FIG. 2

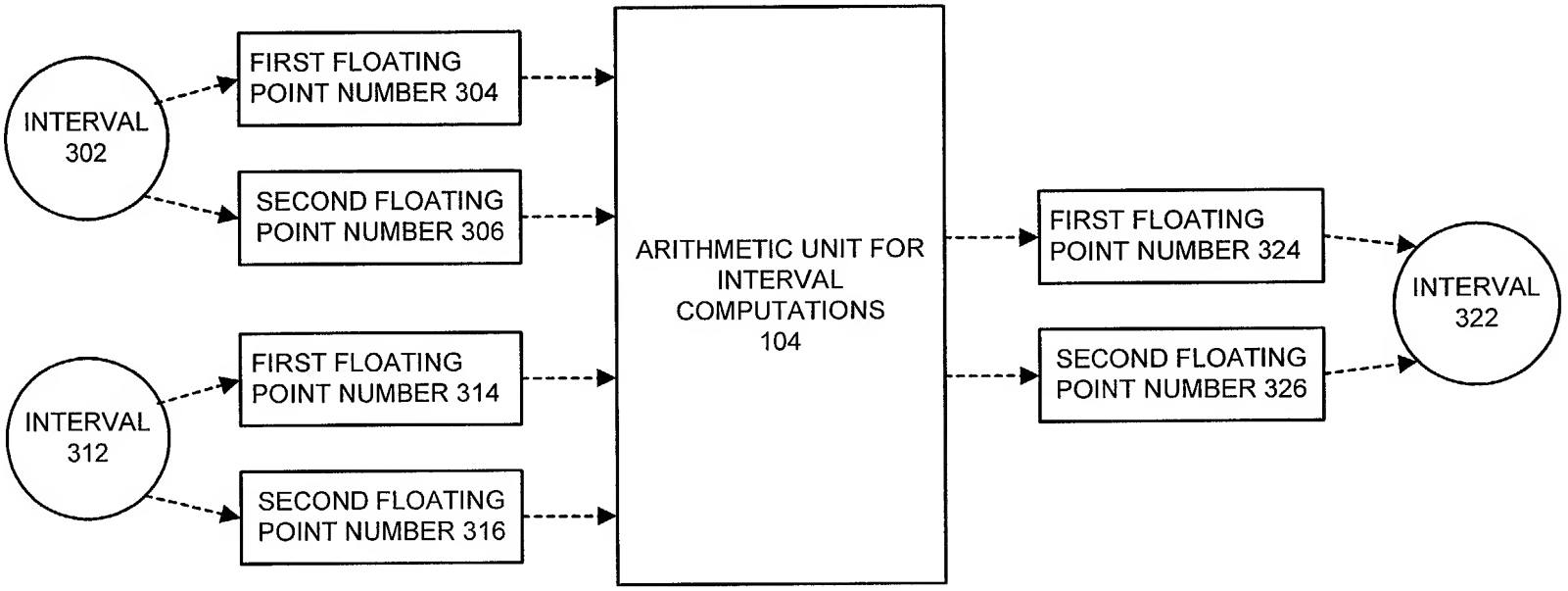
INTERVAL
302
FIRST FLOATING
POINT NUMBER 304
SECOND FLOATING
POINT NUMBER 306
INTERVAL
312
FIRST FLOATING
POINT NUMBER 314
SECOND FLOATING
POINT NUMBER 316
ARITHMETIC UNIT FOR
INTERVAL
COMPUTATIONS
104
FIRST FLOATING
POINT NUMBER 324
SECOND FLOATING
POINT NUMBER 326
INTERVAL
322

FIG. 3

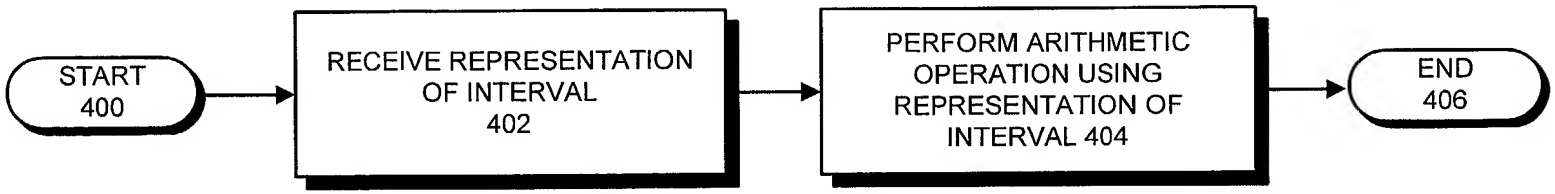
START
400
RECEIVE REPRESENTATION
OF INTERVAL
402
PERFORM ARITHMETIC
OPERATION USING
REPRESENTATION OF
INTERVAL 404
END
406

FIG. 4

$$X \equiv \left[\underline{x}, \bar{x}\right] \equiv \left\{x \in \Re^* \middle| \underline{x} \le x \le \bar{x}\right\}$$

$$Y \equiv \left[\underline{y}, \bar{y}\right] \equiv \left\{y \in \Re^* \middle| \underline{y} \le y \le \bar{y}\right\}$$

(1) $$X + Y = \left[\downarrow \underline{x} + \underline{y}, \uparrow \bar{x} + \bar{y}\right]$$

(2) $$X - Y = \left[\downarrow \underline{x} - \bar{y}, \uparrow \bar{x} - \underline{y}\right]$$

(3) $$X \times Y = \left[\min\left(\downarrow \underline{x} \times \underline{y}, \underline{x} \times \bar{y}, \bar{x} \times \underline{y}, \bar{x} \times \bar{y}\right), \max\left(\uparrow \underline{x} \times \underline{y}, \underline{x} \times \bar{y}, \bar{x} \times \underline{y}, \bar{x} \times \bar{y}\right)\right]$$

(4) $$X / Y = \left[\min\left(\downarrow \underline{x} / \underline{y}, \underline{x} / \bar{y}, \bar{x} / \underline{y}, \bar{x} / \bar{y}\right), \max\left(\uparrow \underline{x} / \underline{y}, \underline{x} / \bar{y}, \bar{x} / \underline{y}, \bar{x} / \bar{y}\right)\right], \text{ if } 0 \notin Y$$

$$X / Y \subseteq \Re^*, \text{ if } 0 \in Y$$

FIG. 5

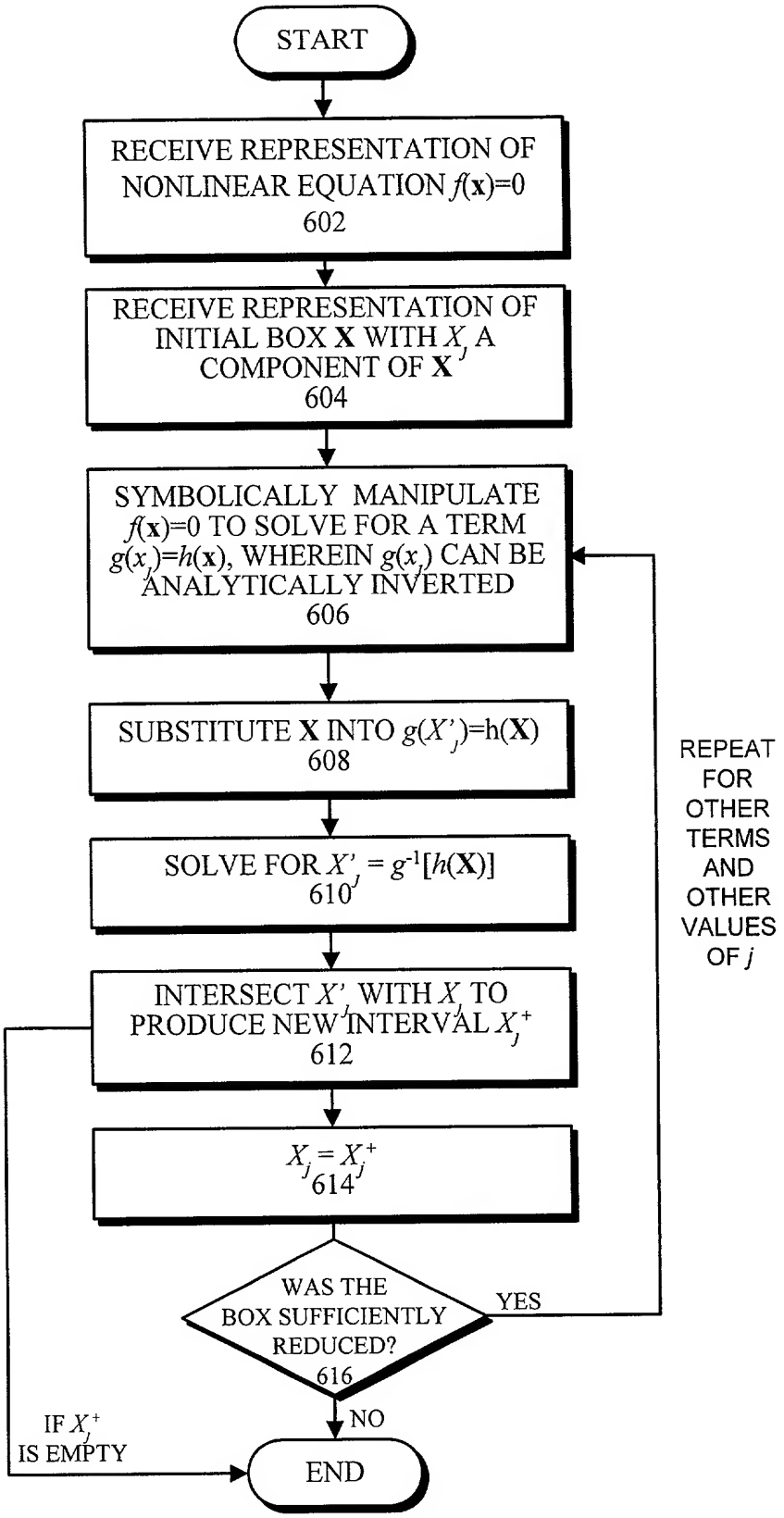
START
RECEIVE REPRESENTATION OF NONLINEAR EQUATION $f(\mathbf{x})=0$
602
RECEIVE REPRESENTATION OF INITIAL BOX $\mathbf{X}$ WITH X_j A COMPONENT OF $\mathbf{X}$
604
SYMBOLICALLY MANIPULATE $f(\mathbf{x})=0$ TO SOLVE FOR A TERM $g(x_j)=h(\mathbf{x})$, WHEREIN $g(x_j)$ CAN BE ANALYTICALLY INVERTED
606
SUBSTITUTE $\mathbf{X}$ INTO $g(X'_j)=h(\mathbf{X})$
608
SOLVE FOR $X'_j = g^{-1}[h(\mathbf{X})]$
610
INTERSECT X'_j WITH X_j TO PRODUCE NEW INTERVAL X_j^+
612
$X_j = X_j^+$
614
WAS THE BOX SUFFICIENTLY REDUCED?
616
YES
NO
IF X_j^+ IS EMPTY
END
REPEAT FOR OTHER TERMS AND OTHER VALUES OF j

FIG. 6

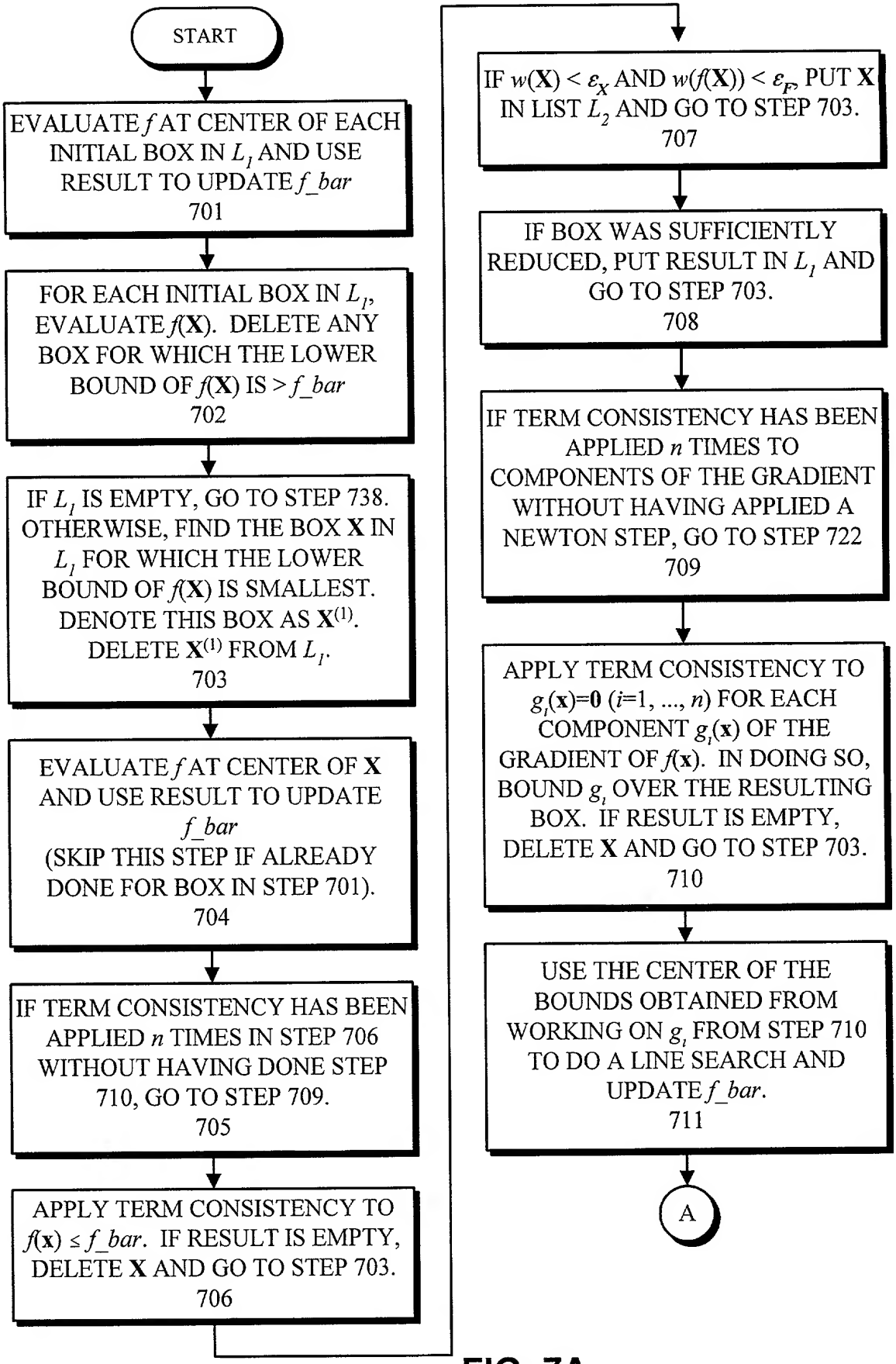
START
EVALUATE f AT CENTER OF EACH INITIAL BOX IN L_1 AND USE RESULT TO UPDATE f_bar
701
FOR EACH INITIAL BOX IN L_1, EVALUATE $f(\mathbf{X})$. DELETE ANY BOX FOR WHICH THE LOWER BOUND OF $f(\mathbf{X})$ IS > f_bar
702
IF L_1 IS EMPTY, GO TO STEP 738. OTHERWISE, FIND THE BOX $\mathbf{X}$ IN L_1 FOR WHICH THE LOWER BOUND OF $f(\mathbf{X})$ IS SMALLEST. DENOTE THIS BOX AS $\mathbf{X}^{(1)}$. DELETE $\mathbf{X}^{(1)}$ FROM L_1.
703
EVALUATE f AT CENTER OF $\mathbf{X}$ AND USE RESULT TO UPDATE f_bar (SKIP THIS STEP IF ALREADY DONE FOR BOX IN STEP 701).
704
IF TERM CONSISTENCY HAS BEEN APPLIED n TIMES IN STEP 706 WITHOUT HAVING DONE STEP 710, GO TO STEP 709.
705
APPLY TERM CONSISTENCY TO $f(\mathbf{x}) \le f_bar$. IF RESULT IS EMPTY, DELETE $\mathbf{X}$ AND GO TO STEP 703.
706
IF $w(\mathbf{X}) < \varepsilon_X$ AND $w(f(\mathbf{X})) < \varepsilon_F$, PUT $\mathbf{X}$ IN LIST L_2 AND GO TO STEP 703.
707
IF BOX WAS SUFFICIENTLY REDUCED, PUT RESULT IN L_1 AND GO TO STEP 703.
708
IF TERM CONSISTENCY HAS BEEN APPLIED n TIMES TO COMPONENTS OF THE GRADIENT WITHOUT HAVING APPLIED A NEWTON STEP, GO TO STEP 722
709
APPLY TERM CONSISTENCY TO $g_i(\mathbf{x})=\mathbf{0}$ (i=1, ..., n) FOR EACH COMPONENT $g_i(\mathbf{x})$ OF THE GRADIENT OF $f(\mathbf{x})$. IN DOING SO, BOUND g_i OVER THE RESULTING BOX. IF RESULT IS EMPTY, DELETE $\mathbf{X}$ AND GO TO STEP 703.
710
USE THE CENTER OF THE BOUNDS OBTAINED FROM WORKING ON g_i FROM STEP 710 TO DO A LINE SEARCH AND UPDATE f_bar.
711
A

FIG. 7A

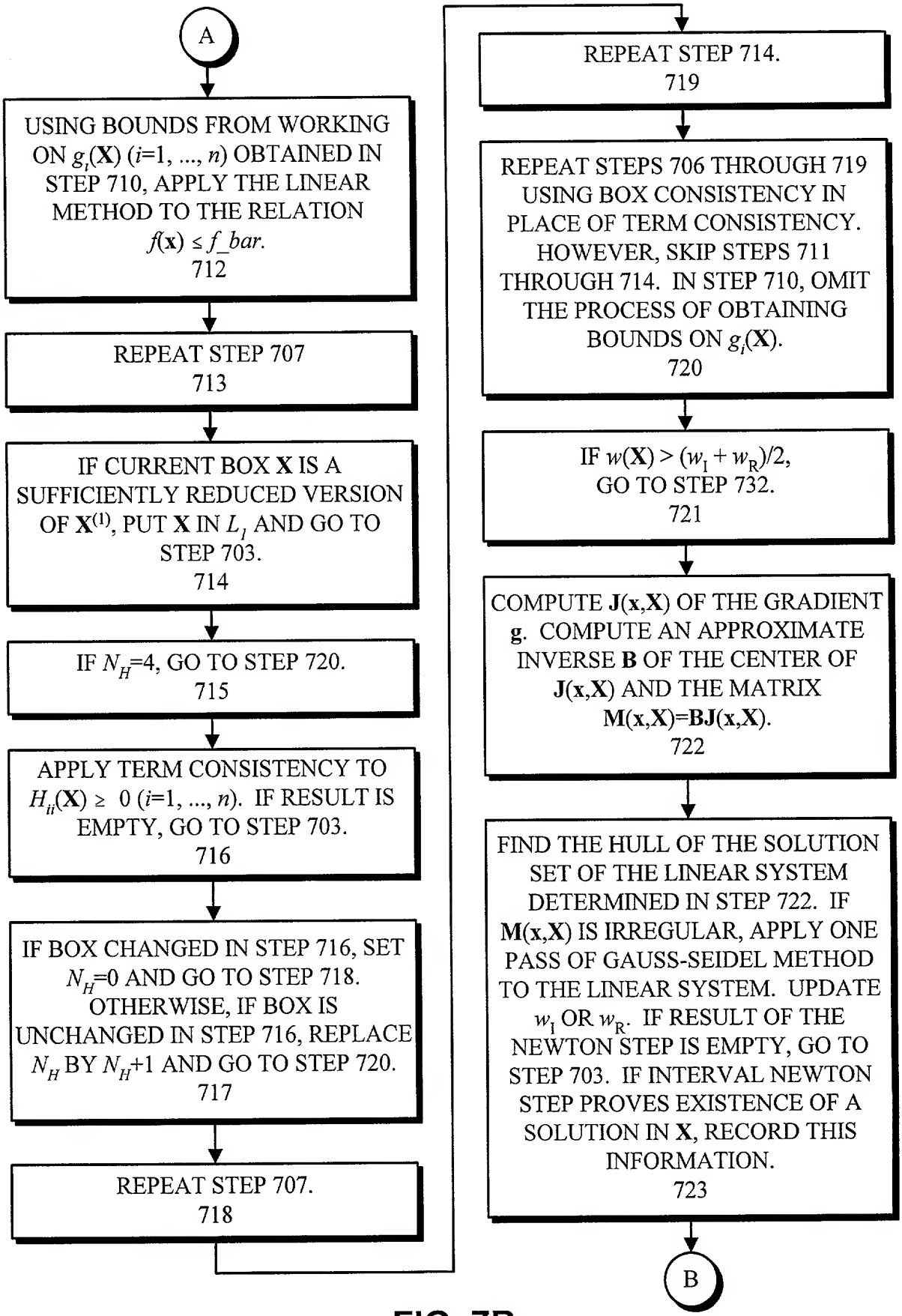

FIG. 7B

EP 0 704 809 A2

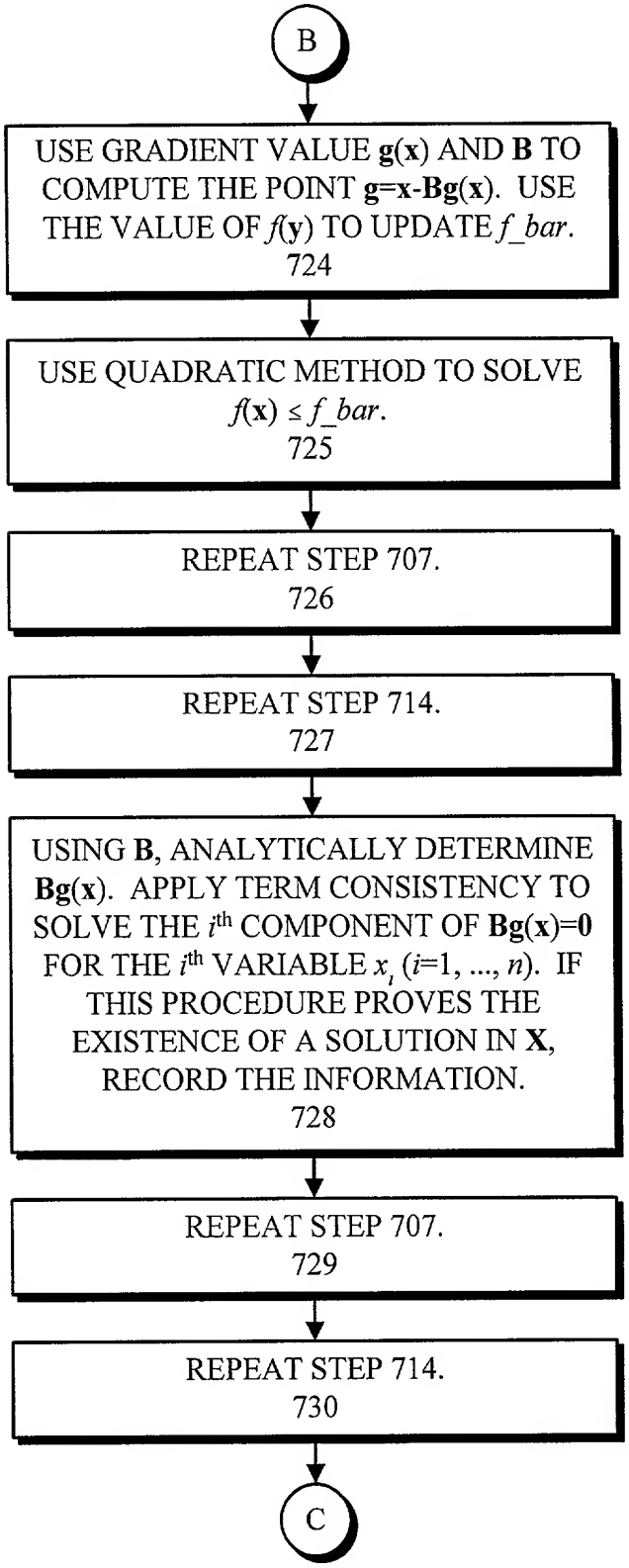
B
USE GRADIENT VALUE g(x) AND B TO COMPUTE THE POINT g=x-Bg(x). USE THE VALUE OF f(y) TO UPDATE f_bar.
724
USE QUADRATIC METHOD TO SOLVE f(x) ≤ f_bar.
725
REPEAT STEP 707.
726
REPEAT STEP 714.
727
USING B, ANALYTICALLY DETERMINE Bg(x). APPLY TERM CONSISTENCY TO SOLVE THE i^th COMPONENT OF Bg(x)=0 FOR THE i^th VARIABLE x_i (i=1, ..., n). IF THIS PROCEDURE PROVES THE EXISTENCE OF A SOLUTION IN X, RECORD THE INFORMATION.
728
REPEAT STEP 707.
729
REPEAT STEP 714.
730
C

FIG. 7C

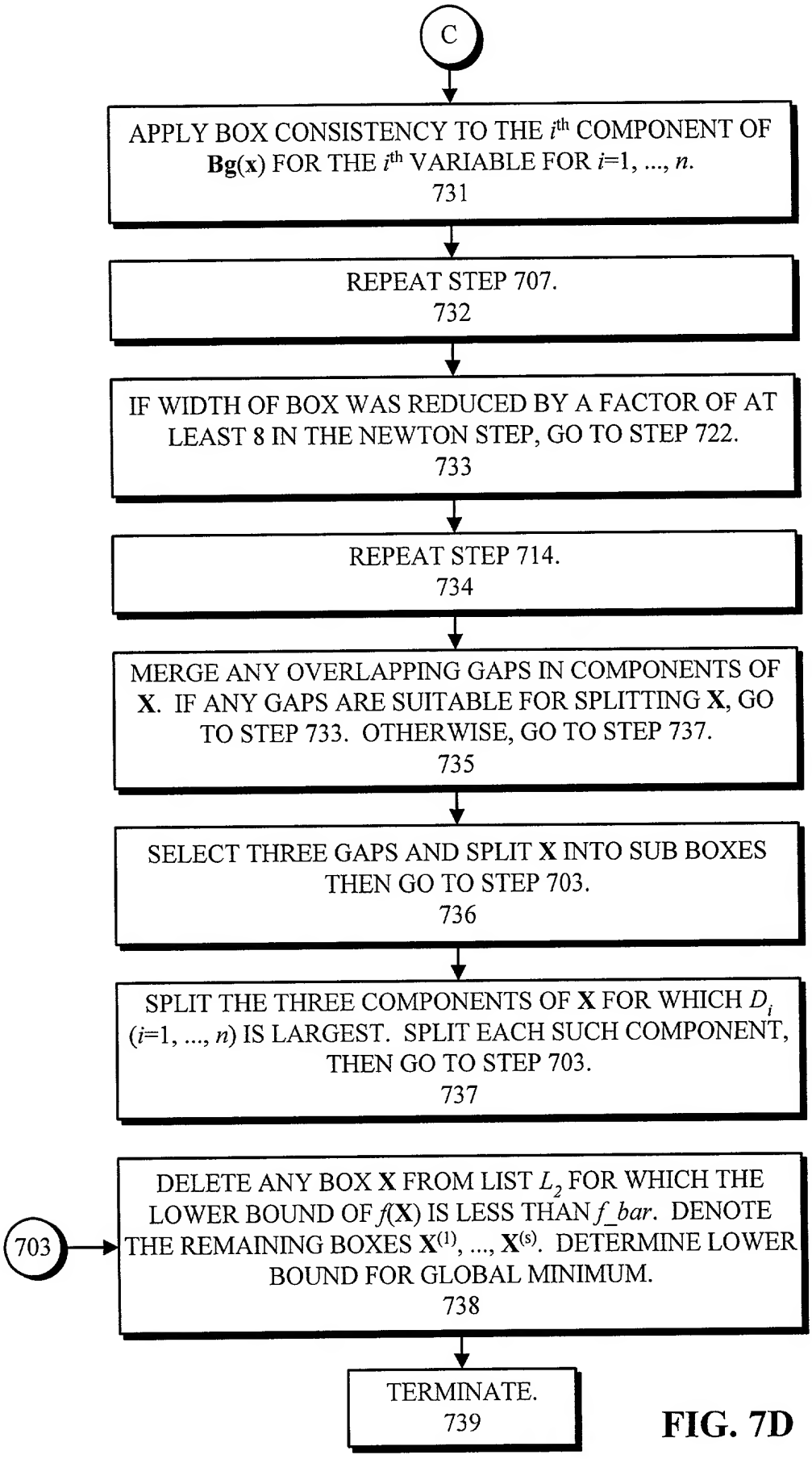
C
APPLY BOX CONSISTENCY TO THE i^{th} COMPONENT OF **Bg(x)** FOR THE i^{th} VARIABLE FOR i=1, ..., n.
731
REPEAT STEP 707.
732
IF WIDTH OF BOX WAS REDUCED BY A FACTOR OF AT LEAST 8 IN THE NEWTON STEP, GO TO STEP 722.
733
REPEAT STEP 714.
734
MERGE ANY OVERLAPPING GAPS IN COMPONENTS OF **X**. IF ANY GAPS ARE SUITABLE FOR SPLITTING **X**, GO TO STEP 733. OTHERWISE, GO TO STEP 737.
735
SELECT THREE GAPS AND SPLIT **X** INTO SUB BOXES THEN GO TO STEP 703.
736
SPLIT THE THREE COMPONENTS OF **X** FOR WHICH D_i (i=1, ..., n) IS LARGEST. SPLIT EACH SUCH COMPONENT, THEN GO TO STEP 703.
737
703
DELETE ANY BOX **X** FROM LIST L_2 FOR WHICH THE LOWER BOUND OF f(**X**) IS LESS THAN f_bar. DENOTE THE REMAINING BOXES $\mathbf{X}^{(1)}$, ..., $\mathbf{X}^{(s)}$. DETERMINE LOWER BOUND FOR GLOBAL MINIMUM.
738
TERMINATE.
739

FIG. 7D